

Section 5.5

Definition of Exponential Function to Base a : If a is a positive real number ($a \neq 1$) and x is any real number, then the **exponential function to the base a** is denoted by a^x and is defined by

$$a^x = e^{(\ln a)x}.$$

If $a = 1$, then $y = 1^x = 1$ is a constant function.

Definition of Logarithmic Function to Base a : If a is a positive real number ($a \neq 1$) and x is any positive real number, then the **logarithmic function to the base a** is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{1}{\ln a} \ln x.$$

Properties of Inverse Functions

1. $y = a^x$ if and only if $x = \log_a y$
2. $a^{\log_a x} = x$, for $x > 0$.
3. $\log_a a^x = x$, for all x

Derivatives for Bases other than e : Let a be a positive real number ($a \neq 1$), and let u be a differentiable function of x .

1. $\frac{d}{dx} [a^x] = (\ln a)a^x$
2. $\frac{d}{dx} [a^u] = (\ln a)a^u \frac{du}{dx}$
3. $\frac{d}{dx} [\log_a x] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx} [\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$

1) The half-life of radium is 1602 years. If a sample contains 5 grams of radium, how much will be left after 700 years?

2) Solve for x in each equation.

a) $4^x = \frac{1}{64}$

b) $\log_3 x = -2$

3) Find the derivative of each function.

a) $y = 5^x$

b) $y = 4^{6x^2}$

c) $y = \log_5 \tan x$

4) Find $\int 3^x dx$

5) Find the following.

a) $\frac{d}{dx} [\pi^x]$

b) $\frac{d}{dx} [x^\pi]$

c) $\frac{d}{dx} [\pi^\pi]$

d) $\frac{d}{dx} [(2x)^{2x}]$